

Phase transition in linear sigma model and disoriented chiral condensate

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We have investigated the phase transition and disoriented chiral condensate domain formation in linear sigma model. Solving the equation of motion for the sigma model fields in contact with a heat bath, we have shown that the fields undergo phase transition above a certain temperature (T_c). It was also shown that when the fields thermalised at temperature above T_c are cooled down sufficiently rapidly, disoriented chiral condensate domains are formed quite late in the evolution.

25.75.+r, 12.38.Mh, 11.30.Rd

The possibility of forming disoriented chiral condensate (DCC) in relativistic heavy ion collisions has generated considerable research activities in recent years. The idea was first proposed by Rajagopal and Wilczek [1–4]. They argued that for a second order chiral phase transition, the chiral condensate can become temporarily disoriented in the nonequilibrium conditions encountered in heavy ion collisions. As the temperature drops below T_c , the chiral symmetry begins to break by developing domains in which the chiral field is misaligned from its true vacuum value. The misaligned condensate has the same quark content and quantum numbers as do pions and essentially constitute a classical pion field. The system will finally relaxes to the true vacuum and in the process can emit coherent pions. Since the disoriented domains have well defined isospin orientation, the associated pions can exhibit novel centauro-like [5–8] fluctuations of neutral and charged pions [9–12].

Most dynamical studies of DCC have been based on the linear sigma model, in which the chiral degrees of freedom are described by the real O(4) field $\Phi = (\sigma, \vec{\pi})$, with the Lagrangian,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{\lambda}{4}(\Phi^2 - f_\pi^2)^2, \quad (1)$$

where λ is a positive coupling constant and f_π is the pion decay constant. At finite temperature, to leading order in λ , the thermal fluctuations $\langle \delta\phi^2 \rangle$ of the pions and σ -mesons do generate an effective Hartree type dynamical mass giving rise to an effective temperature dependent potential $\lambda T^2/2$ [13]. Resulting chiral phase transition is compatible with the expectations of lattice gauge QCD calculations [14]. One generally introduces a finite symmetry breaking term h_σ to take into account the finite pion mass. However, at present, we are ignoring such terms, as our interest is to investigate chiral phase transition and subsequent disoriented chiral condensate domain formation. With symmetry breaking term, there is no exact phase transition.

Aim of the present letter is to study the influence of external source on the phase transition and subsequent DCC domain formation when the chiral symmetric phase relaxes back to symmetry broken phase. Indeed, in a heavy ion collision, while it is possible that in a certain region chiral symmetry is restored, that region must be in contact with some environment i.e. background. Exact nature of the environment is difficult to determine but presumably it will be consists of mesons and hadron (pions, nucleon etc.). Recognising the uncertainty in the exact nature of the environment, we choose to represent it by a white noise source, i.e. a heat bath. To be consistent with fluctuation-dissipation theorem, we also include a dissipative term in the equation of motion for the sigma model fields. Thus we are studying essentially Langevin equation for the O(4) fields. Recently it has been shown that in the ϕ^4 theory, hard modes can be integrated out on a two loop basis resulting in a Langevin type equations for the soft modes [15,16], there by justifying our approach. Langevin equation for O(4) fields have been used by several authors [17,18] to study the interplay of friction and white noise in disoriented chiral condensate formation. We thus propose to study following Langevin equation,

$$[\frac{\partial^2}{\partial \tau^2} + (\frac{1}{\tau} + \eta) \frac{\partial}{\partial \tau} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{1}{\tau^2} \frac{\partial^2}{\partial Y^2} + \lambda(\Phi^2 - f_\pi^2 - T^2/2)]\Phi = \zeta(\tau, x, y, Y) \quad (2)$$

where we have used proper time (τ) and rapidity (Y), which are the appropriate coordinates for heavy ion scattering. In eq. 2 η is the friction. As told earlier, the environment or the heat bath (ζ) was represented as a white noise source with zero average and correlation as demanded by fluctuation-dissipation theorem,

$$\langle \zeta(\tau, x, y, Y) \rangle = 0 \quad (3a)$$

$$\int \langle \zeta_a(\tau_1, x_1, y_1, Y_1) \zeta_b(\tau_2, x_2, y_2, Y_2) \rangle d^4x = 2T\eta\delta_{ab} \quad (3b)$$

Set of partial differential eqs. 2 were solved on a 32^3 lattice using a lattice spacing of 1 fm, thus cutting off modes with momenta >200 MeV. To show that the model undergoes phase transition we keep the heat bath at fixed temperature T and evolve the fields for sufficiently long time (30 fm) such that equilibrium is reached. Fields thermalised at higher temperature will be more randomized than fields thermalised at lower temperature. At low temperature, the randomisation will not be complete and $\langle \sigma \rangle$ will have definite non-zero value. Above the critical temperature (T_c), randomisation will be complete and $\langle \sigma \rangle$ will be zero, indicating restoration of symmetry.

Solving eqn.2 require initial conditions (ϕ and $\dot{\phi}$). We distribute the initial fields according to a random Gaussian with,

$$\langle \sigma \rangle = f(r)f_\pi \quad (4a)$$

$$\langle \pi_i \rangle = 0 \quad (4b)$$

$$\langle \sigma^2 \rangle - \langle \sigma \rangle^2 = \langle \pi_i^2 \rangle - \langle \pi_i \rangle^2 = f_\pi^2/4f(r) \quad (4c)$$

$$\langle \dot{\sigma} \rangle = \langle \dot{\pi}_i \rangle = 0 \quad (4d)$$

$$\langle \dot{\sigma}^2 \rangle = \langle \dot{\pi}^2 \rangle = f_\pi^2 f(r) \quad (4e)$$

The interpolation function

$$f(r) = [1 + \exp(r - r_0)/\Gamma)]^{-1} \quad (5)$$

separates the central region from the rest of the system. We have taken $r_0=11$ fm and $\Gamma=0.5$ fm. The initial field configurations corresponds to quench like condition [1] but it is important to note that the field configuration at equilibrium will be independent of initial configuration. Equilibrium value depend on the heat bath only. We have verified this in our code. The other parameter of the model is the friction (η). In the present paper, we use $\eta = \eta_\pi + \eta_\sigma$ and for η_π and η_σ use values as calculated by Rischke [19] but once again, its precise value is not of importance here, as we are looking for fields at equilibrium. Friction determines the rate of approach to equilibrium. This aspect of equilibration was also verified.

The condensate value of the σ field can be considered as the order parameter for the phase transition. If there is a phase transition at T_c , the order parameter should *exactly* vanish for temperatures $\geq T_c$, while below T_c it will have nonzero values [20]. We calculate the order parameter at the end of the evolution as,

$$\langle \sigma \rangle = \frac{\int \sigma dx dy dY}{\int dx dy dY}. \quad (6)$$

In fig.1, we have shown the variation of the order parameter with temperature. At low temperature, as expected, order parameter is around $f_\pi=92$ MeV. With increasing temperature, $\langle \sigma \rangle$ decreases and become zero around $T_c=120$ MeV, indicating restoration of chiral symmetry. It remain zero at higher temperatures also. The behaviour of the σ condensate corresponds to a second order phase transition. The critical temperature $T_c=120$ MeV is also in agreement with mean field calculations.

Chirally symmetric phase at high temperature will roll back to symmetry broken phase as the system cools and temperature drops below the transition temperature. As told earlier, it is has been conjectured [1] that domain like structure with definite isospin content may emerge during the roll down period. Numerical simulations of linear sigma model indicate that with quench like initial condition, domain like configurations do indeed emerge as the chirally symmetric phase roll back to symmetry broken state [21]. However, in heavy ion collision, quench is not a natural initial condition. $\langle \phi \rangle$ and $\langle \dot{\phi} \rangle$ are in a configuration appropriate for high temperature but that of $\langle \phi^2 \rangle$ and $\langle \dot{\phi}^2 \rangle$ are characteristic of a lower temperature. On the contrary, thermalised fields are better suited to mimic initial conditions that may arise in heavy ion collisions. Here, $\langle \phi \rangle$, $\langle \dot{\phi} \rangle$ as well as $\langle \phi^2 \rangle$ and $\langle \dot{\phi}^2 \rangle$ are in a configuration appropriate for high temperature.

To see whether domain like structure emerges or not with more appropriate initial condition like the thermalised fields, we did a demonstrative calculation. The fields were thermalised at $T=200$ MeV. We then allow the heat bath to cool and follow the evolution of the thermalised fields. Evolution of the thermalised fields now will depend on the exact nature of the friction, however we choose to use the same friction as before. We assume the following cooling law for the heat bath,

$$T(t) = T_0 \frac{1}{t^n} \quad (7)$$

with $n=1$ (appropriate for 3d scaling expansion). Assuming that the number density is proportional to the square of the amplitude, at each lattice point, we calculate the neutral to total pion ratio according to,

$$R(x, y, Y) = \frac{N_{\pi_0}}{N_{\pi_1} + N_{\pi_2} + N_{\pi_0}} \quad (8)$$

Very large or small value of the ratio, over an extended spatial zone will be definite indication of disoriented chiral condensate domain formation. At this point we would like to note that contour plot of one single component of the pion field as has been shown in ref. [21,22] do not necessarily indicate domain formation.

In fig.2 we have shown contour plot of the ratio R in xy plane at rapidity $Y=0$. The initial random distribution (panel a) do not show any domain like structure with very high or low value of the ratio. After the thermalisation of the fields at $T=200$ MeV, no domain like structure is evident (panel b). Some small zones with large value of R are formed, but they are just thermal fluctuations. As will be shown below, thermal fields have larger correlation than random fields in quenched initial condition. Small zones with large r are manifestation of that correlation. Large domain like structure with high/low value of the ratio R starts to emerge after 10-15 fm of evolution and cooling. With time domain like structure grow. At other rapidities also, similar behaviour is seen. Fig.2, clearly demonstrate that with appropriate cooling law, multiple disoriented chiral condensate domains with different isospin orientations can form as the chirally symmetric phase roll back to broken phase. However domain formation occur quite late in the evolution. With the scaling cooling law, by the time domain like structures emerge, the system is cooled to ~ 20 MeV. It is doubtful whether in heavy ion collision, system can be allowed to be cooled to this extent. Current wisdom is that the hadrons freeze-out around 100-160 MeV. With this reservation in mind, it may be said that even with thermal fields domains of disoriented chiral condensate can form.

Above behaviour is confirmed from the correlation study also. We define a correlation function at rapidity Y as [21],

$$C(r, \tau) = \frac{\sum_{i,j} \pi(i) \dot{\pi}(j)}{\sum_{i,j} |\pi(i)| |\pi(j)|} \quad (9)$$

where the sum is taken over those grid points i and j such that the distance between i and j is r . In fig. 3, we have shown the evolution of the correlation function at rapidity $Y=0$. Initially there is no correlation length beyond the lattice spacing of 1 fm. After thermalisation, the correlation is increased marginally. At the thermalisation, the fields are at the minimum of the potential, consequently develops some correlation. Correlation do not increase till 10 fm of evolution. At later time, long range correlation develops, increasing with time. At 20 and 30 fm, pions separated by large distance are correlated.

If a single DCC domain is formed in heavy ion collision, it can be easily detected. Probability to obtain a particular fraction f of (say) neutral pion from a single domain is $P(f) = 1/2\sqrt{f}$ [1]. However, our simulation indicate that a few numbers of domains with different isospin orientations are formed. Naturally the resultant distribution will not be $1/\sqrt{f}$ type. In fig.4, f distribution for 100 events, at zero rapidity are shown. Density of pions at rapidity Y was calculated by integrating over the space-time as,

$$N_{\pi}(Y) = \int \pi^2 \tau d\tau dx dy \quad (10)$$

If the thermalised fields are evolved upto $\tau=10$ fm, then the f distribution is sharply peaked around the isospin symmetric average value of $1/3$. At early stage of evolution, as indicated in fig.2 and 3, long range correlations or domain like structure are not developed in individual events. If the fields are evolved upto 20 or 30 fm, then there is definite domain like formation and also long range correlation is developed, in individual events. The f distribution still has average $1/3$, but the width is increased. This indicate that large fluctuations in the ratio r from event to event is expected when there is DCC like phenomena. Experimentally, those events for which neutral to total pion ratio shows considerable fluctuations will be promising candidate for DCC events.

In summary, we have studied the phase transition and disoriented chiral condensate formation in linear sigma model in presence of background. The background was represented by a white noise source i.e. a heatbath. Evolving the fields in contact with the heat bath for sufficiently long time such that fields are thermalised, it was shown the the model undergoes 2nd order phase transition around $T_c=120$ MeV. It was also shown that if the thermalised fields are cooled down sufficiently rapidly, multiple disoriented chiral condensate domains are formed. Long range correlation also develops. However, even with sufficient fast cooling, with thermalised fields, domains or long range correlations develop quite late in the evolution (after 10-15 fm of cooling, when the fields are cooled to 20 MeV or so). It is doubtful whether in heavy ion collision, the hadronic fluid can be allowed to evolve and cool for such long time. It is possible that they undergoes freeze-out much earlier. Then, disoriented chiral condensate domains will not be formed. We have also shown that with multiple domain formation, probability distribution do not follow the $1/\sqrt{f}$ type of law, rather, it is a Gaussian with average at the isospin symmetric value of $1/3$, but with increased fluctuations. It is suggested that one should look into events with large fluctuations of the neutral to pion ratio around the isospin symmetric value of $1/3$.

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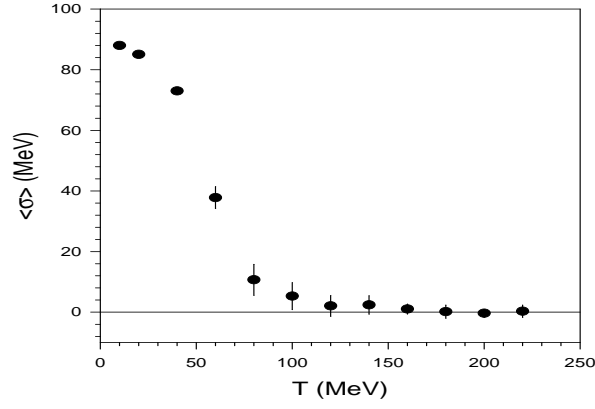


FIG. 1. Equilibrium value of the order parameter as a function of the temperature.

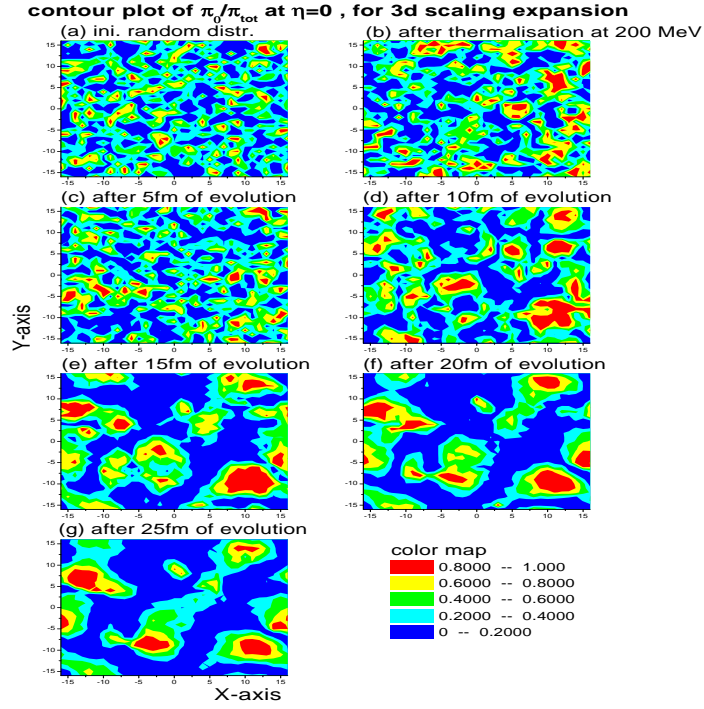


FIG. 2. contour plot of the neutral to total pion ratio at rapidity $Y=0$ at different stages of evolution.

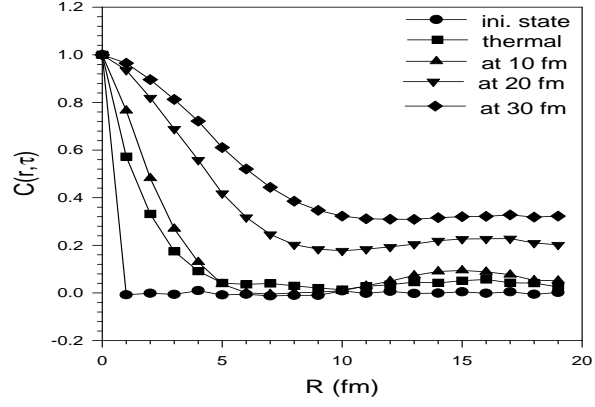


FIG. 3. Evolution of the correlation function at rapidity $Y=0$.

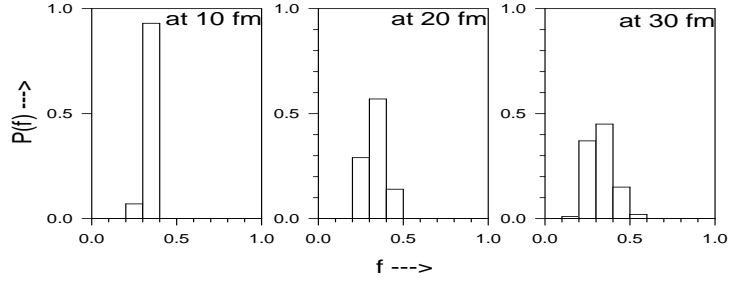


FIG. 4. Probability distribution for the neutral to total pion ratio at rapidity $Y=0$, for 100 events at different stages of evolution